RESEARCH ON THE DIGITAL MIXED DENOISING TECHNIQUES FOR ACCELERATING MONTE CARLO SIMULATIONS

RADIATION PROTECTION

KEYWORDS: *dose distribution, Monte Carlo method, mixture filter*

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The Monte Carlo method is a stochastic statistic algorithm. It is one of the most accurate dose calculation methods, but its application in clinic is limited because of the long computation time. Generally, to accelerate Monte Carlo simulation and reduce stochastic noise, a digital filtering technique is used to smooth a rough dose distribution (includes evident noise) to a satisfied one. Different types of filters have been applied, such as Gaussian filters, Savitzky-Golay filters, etc., but the ability of a single filtering filter is limited. Therefore, a hybrid filter combining those filtering techniques was used. Two types of mixture methods—parallel and cascade—with three-dimensional Gaussian and Savitzky-Golay filters were researched. In addition, a method that simplifies the mixture filter structure using an equivalent convolution kernel based on convolution theory was introduced. With simulation data from a standard phantom, the rough dose distributions and the dose distribution smoothed by the two types of mixture filters were compared with that of the "benchmark" one. Test results showed that the two types of mixture filters can suppress much of the noise added in Monte Carlo dose distributions and enhance its visualization. As for the research's test cases, the filtering effect of the cascade mixture filter was slightly better than that of the parallel mixture filter. Filter combinations can provide favorable filtering effects. The filtering effects of different mixture methods are not uniform. The cascade mixture filter has a better filtering effect than the parallel mixture filter.

I. PURPOSE

The Monte Carlo (MC) method has many advantages, such as high precision, independence of dimensionality, and ability to deal with an inhomogeneous medium. The MC method is recognized as the most powerful tool for simulating particle transport in the field of radiation therapy medicine. However, the MC method has been used only as a validation tool for an advanced therapy plan system because of the slow convergence, the long computation time, and the unavoidable stochastic fluctuation (i.e., noise). The stochastic fluctuation of the MC method simulation particle transport primarily affects computation precision and speed. The greater stochastic fluctuation not only affects the visualization effect of the dose distributions but also cannot satisfy the clinical application dose requirement because of low precision. The computation time of the

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MC method must be long enough to obtain highly accurate results.

To apply MC simulation results to clinical practice, Deasy et al.[1,2] proposed the denoising method using digital filtering techniques to accelerate the MC dose distributions, i.e., reducing stochastic noise, cutting down simulation time, and making computation precise enough to satisfy the requirements of clinical practice. Deasy et al.[1,2] discussed the applications of two-dimensional (2D) Gaussian and the local least-squares filters in 2D dose distributions. Krumm[3] applied the Savitzky-Golay (SG) filters to smoothing the 2D images. Lin and Luo[4,5] applied the Gaussian and SG filters to the three-dimensional (3D) denoising of MC dose distributions.

This work applies signal-processing technology to mixing two types of filters based on 3D Gaussian and SG filters. There are two types of mixture methods: parallel mixture and cascade mixture. Test results show that the mixture filters can greatly suppress the noise added in MC dose distributions and enhance visualization. The filtering effect of the cascade mixture filter is slightly better than that of the parallel mixture filter.

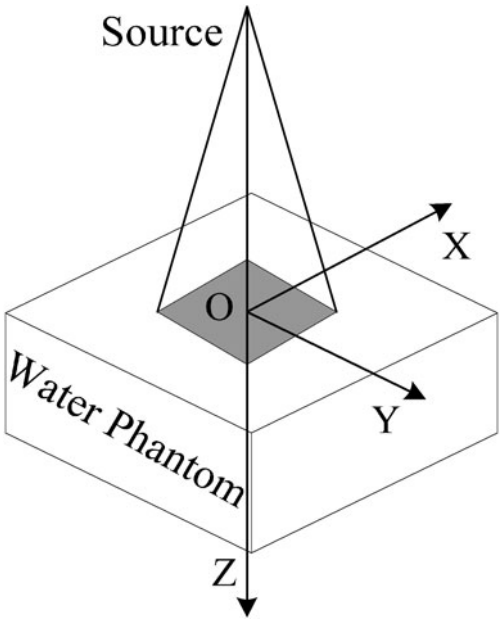

Fig. 1. The reference frame.

II. METHODS AND MATERIALS

II.A. Test Case

A point source with a 6-MV Mohan energy spectrum incident on a 30- × 30- × 30-cm water phantom is simulated using the MC program EGSnrc/DOSXYZnrc. A dose distribution of 5×10^7 particle histories is used as the rough dose, and a dose distribution of 1×10^9 particle histories is used as the benchmark. The grid size is 5 × 5 × 2 mm, and the voxel number is 540 000. The source skin distance is 100 cm, and the field size is 10 × 10 cm. In the reference frame (Fig. 1), the origin O of the 3D coordinates is set on the symmetry center of the water phantom surface. The photon beam perpendicularly irradiates on the surface center of the water phantom, and its symmetry axis Z axis (i.e., the depth) goes through the origin O. The positive direction of the Z axis is the same as the incident direction. The coordinate plane XOY is set on the water phantom surface, in addition, where X and Y axes, respectively, plumb to the adjacent border of the surface rectangle. The program language is C++.

This work chooses 3D Gaussian and SG filters. For the detailed principles and their mathematical models, see Refs. 2 and 3, where the convolution kernel window size of the 3D Gaussian filter is 3 × 3 × 3 and the convolution kernel window size of the SG filter is 5 × 5 × 3 with order 3.

II.B. Signal-Processing Technology

Madansky (1988) proposed the variance stabilizing transformation in order to use the filtering techniques in MC dose distributions. Deasy et al.[1] have proved that the transformation is applicable for MC dose distributions according to mathematics derivations and experiments. Suppose the rough dose with noise is d. Before applying filtering techniques to d, d should be transformed so that the variance in rough doses is nearly constant. The transformation is

$$D = \sqrt{d} \quad ; \tag{1}$$

whereafter, apply 3D filters into the transformed dose D to obtain filtered dose D'. Finally, square the filtered dose D' to obtain final dose d'.

When filtering MC rough dose distributions, a 3D convolution kernel is needed. The filtering effects of different convolution kernels are not absolutely uniform. The mathematical model of digital filtering denoising is convolution computation in the spatial domain, and the 3D convolution formula is

$$\begin{aligned} D'(x,y,z) &= D(x,y,z) * h(i,j,k) \\ &= \sum_{i=-(L-1)/2}^{(L-1)/2} \sum_{j=-(M-1)/2}^{(M-1)/2} \sum_{k=-(N-1)/2}^{(N-1)/2} \\ &\quad \times D(x+i, y+j, z+k) \cdot h(i,j,k) \quad , \end{aligned} \tag{2}$$

where $*$ is defined as the convolution operator, and L, M, and N, respectively, correspond to convolution kernel size at X, Y, and Z axes, and all of them are odd. We apply the aforementioned mathematical model to filter the MC dose distributions. According to the convolution computation and signal-processing principles,[6] the MC rough

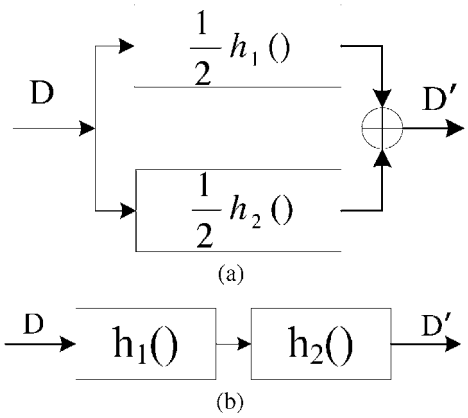

Fig. 2. The system block diagram of (a) the parallel connection and (b) the cascade connection.

dose D can be treated as input signal, and h is the transfer function (unit impulse response) of the signal-processing system; therefore, output signal D' equals the convolution of D and h, i.e., the filtered dose.

The h is a single convolution kernel in Eq. (2). This work uses multiple convolution kernels to construct the mixture filters to further enhance the visualizations of the filtered MC dose distribution. There are many mixture methods, but the basic modes can be classified as parallel connection and cascade connection.

Figures 2a and 2b show the system block diagrams of mixture filters, where h_1 is the SG convolution kernel and h_2 is the Gaussian convolution kernel. Figure 2a is a parallel connection, and Fig. 2b is a cascade connection. This work uses software to implement these types of digital signal-processing systems, which are applied to filter the MC dose distributions.

II.C. The 3D Mixture Filters

The system block diagram of the 3D parallel mixture filter is shown in Fig. 2a. The MC rough dose distribution D is convolved with SG kernel $\frac{1}{2}h_1$ and Gaussian kernel $\frac{1}{2}h_2$, respectively, and then the results of the two filters are summed to obtain filtered dose distribution D'.

The system block diagram of the 3D cascade mixture filter is shown in Fig. 2b. The MC rough dose distribution D is convolved with SG kernel h_1 first, and then the result is convolved with Gaussian kernel h_2. The ultimate result D' is filtered dose distribution.

Generally, h_1 and h_2 also can be replaced with parallel substructure and cascade substructure in the previous mixture structures to form more complicated mixture filters, which might be called the serial and parallel filters. This paper discusses the two types of basic mixture modes.

As implementation of the parallel and cascade mixture filter with software, the cost of program time is approximately equivalent and equals the sum of h_1 and h_2 convolution computation times. According to convolution characteristics, these mixture structures can be simplified and the equivalent convolution kernel can be formed as follows:

$$h_p(\,) = \tfrac{1}{2}h_1(\,) + \tfrac{1}{2}h_2(\,) \qquad (3)$$

$$h_c(\,) = h_1(\,) * h_2(\,)\ , \qquad (4)$$

where kernel h_p is the equivalent convolution of the parallel connection of h_1 and h_2, which equals their sum, and h_c is the equivalent convolution kernel of the cascade connection of h_1 and h_2, which equals their convolution. In this way, the multiple-kernel structures are simplified to the single-kernel structures hp and h_c in Figs. 2a and 2b. Remarkably, equivalent convolution kernels have new window sizes. Suppose the window size of h_1 is $L_1 \times M_1 \times N_1$ and the window size of h_2 is $L_2 \times M_2 \times N_2$. Then, the window size of the equivalent parallel connection is $\mathrm{MAX}(L_1, L_2) \times \mathrm{MAX}(M_1, M_2) \times \mathrm{MAX}(N_1, N_2)$, where MAX is maximum operation. The window size of the equivalent cascade connection is $(L_1 + L_2 - 1) \times (M_1 + M_2 - 1) \times (N_1 + N_2 - 1)$.

III. RESULTS

To quantificationally evaluate the filtering effects of the two types of mixture filters, the root-mean-square deviation (RMSD) analysis is given in Table I at depths of $Z = 0.5$, 1.5 (the depth of buildup region), and 10.5 cm. The RMSD is defined as

$$\mathrm{RMSD} = \sqrt{\frac{\sum_{k=1}^{K}\left[\dfrac{d'_k - bd_k}{bd}\right]^2}{K}} \times 100\%\ , \qquad (5)$$

where bd is the benchmark dose and K is the amount of dose points for evaluation.

The RMSD of the different dose distribution regions are computed respectively to emphasize the dose errors of interested regions. The RMSD indicates the extent (i.e., the dispersion degree) of filtered dose distributions departing from benchmark dose distributions. Low RMSD means filtered dose distributions approximate well to benchmark dose distributions. Table I shows that the mixture filters can, to a large extent, suppress the noise in the MC dose distributions. By comparison, the RMSDs of the parallel mixture filters are lower than those of the cascade mixture filters; i.e., the filtered dose distributions of the cascade mixture filters have less offset from the benchmark dose distributions. This means that the filtering performance of the cascade mixture filter is slightly better than that of the parallel mixture filter.

Figure 3 illustrates the dose distribution contours (have been normalized) at depth $Z = 1.5$ cm after 3D mixed filtering. Figure 3a shows the rough dose

TABLE I

Root-Mean-Square Deviation*

Dose Distributions	Z = 0.5 cm			Z = 1.5 cm (The depth of buildup region)			Z = 10.5 cm		
	All	Field	Outside Penumbra	All	Field	Outside Penumbra	All	Field	Outside Penumbra
Parallel	6.78	1.39	7.19	6.21	0.95	6.57	7.54	1.83	8.07
Cascade	6.62	1.31	7.01	5.98	0.97	6.33	7.32	1.42	7.84
Rough	18.04	2.21	19.34	16.71	2.10	17.71	13.92	2.42	14.93

*In percent.

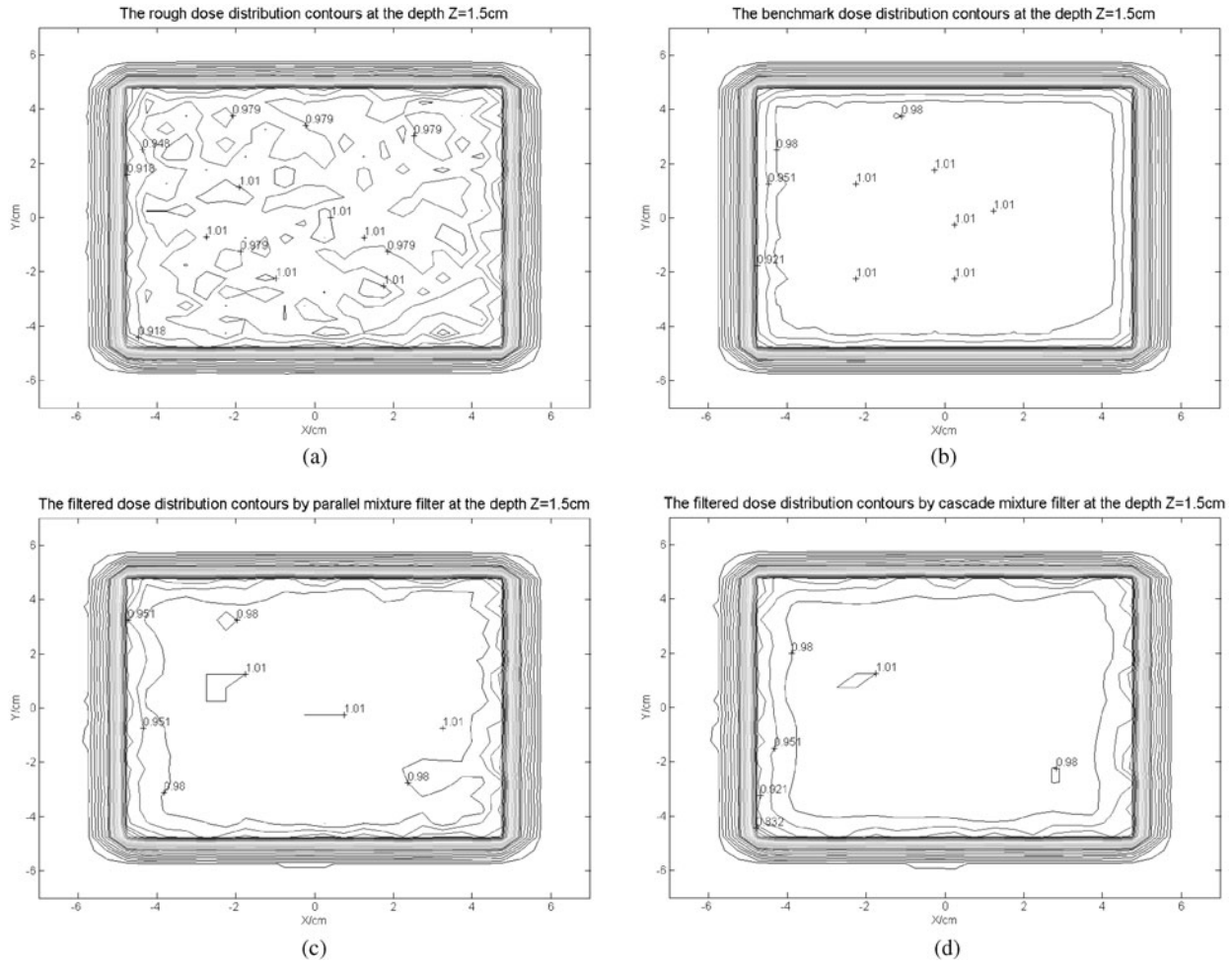

Fig. 3. (a) The rough dose distribution contours and (b) the benchmark dose distribution contours. The dose distribution contours filtered by (c) a parallel mixture filter and (d) a cascade mixture filter.

distribution contours. Figure 3b shows the benchmark dose distribution contours. Figure 3c shows the dose distribution contours filtered by a parallel mixture filter. Figure 3d shows the dose distribution contours filtered by a cascade mixture filter. One can see that the filtering effect of the cascade mixture filter is better than that of the parallel mixture filter by qualitative analysis of Figs. 3a through 3d.

IV. CONCLUSIONS

To make the MC rough dose distributions, whose particle history is less and simulation time is short, satisfy the requirements of clinical practice, the work denoises the rough dose distributions with 3D filters. The 3D Gaussian and SG filters are chosen and mixed, and the two types of mixture methods are chiefly compared. Test results shows that the mixture filters enhance the visualization of the filtered dose distribution. On the whole, the filtering effect of the cascade mixture filter is better than that of the parallel mixture filter. At the same time, this work simplifies the multiple convolution kernels into single convolution kernels and introduces the computation method of equivalent convolution kernels based on convolution characteristics. According to signal and system principles, an arbitrary complex topology structure can be constructed with the basic structures; i.e., arbitrary complex filters can be combined by using the parallel and cascade mixture filters.

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